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Optimal input signal design for parameter estimation in an inertial system with respect to D-efficiency constraints

1 Introduction

The choice of an input signal used for actuation of the system is critical in the task of model building and parameter identification. System identification is the process of constructing an accurate and reliable dynamic mathematical model of the system from observed data and available knowledge. It is a common practice to perturb the system of interest and use the resulting data to build the model [1, 2]. The accuracy of parameter estimates is increased by the use of optimal excitation signals [3, 4].

Particular industries, such as petrochemical and refining industries, rely almost exclusively on system identification as the principal means for obtaining dynamic models for advanced control purposes. The input design problem with respect to the intended model application which is often a control task, has received considerable attention in the last years [5, 6]. It was reported that model development absorbs about 75% of the costs associated with advanced control projects [7]. That is why the input signal used for perturbing the system should by carefully selected.

System identification, in practice, is carried out by perturbing processes or plants under operation. In many industrial applications a plant friendly input signal would be preferred for system identification. Plant friendly identification experiments are those that satisfy plant or operator constraints on experiment duration, input and output amplitudes or input rate [8, 9]. Techniques for synthesising multi-harmonic signals with low crest factors, which are attractive from a plant friendly perspective, have been reported in [2]. It was demonstrated that plant friendliness demands are often in conflict with requirements for good identification [10, 11]. Hence, plant friendly input design is inherently multi-objective in nature.

There have been some reports on multi-objective optimisation based methods, applied to identification and control [12]. However, such an approach to plant-friendly input design with respect to cost function D-efficiency constraint has not been attempted. In the design of optimal inputs for system identification the sensitivity of the state variable to the unknown parameter has been maximised so far. The case study results of optimal input signal design utilising Mayer's canonical formulation of the performance index for the inertial and torsional spring system were presented in [13, 14]. We present here a Mayer's canonical formulation of the performance index for plant friendly identification of an inertial system.

2 Input friendliness factor formulation

The purpose of the current work is to formulate the optimization problem for plant friendly input signal design with respect to D-efficiency constraint. In that way (i.e. by setting such a constraint to control the level of D-optimality loss) we can obtain the most friendly input signal, reducing the rapid changes of the mass or energy inflow to the system.

For a continuous system, the definition of the input friendliness index is as follows [8]

$$\Phi_i = 1 - \frac{\int_0^T (\dot{u}^{\mathrm{T}}(t)\dot{u}(t))dt}{\max\limits_{0\le t\le T} (\dot{u}^{\mathrm{T}}(t)\dot{u}(t))} . \tag{1}$$

The above plant friendliness factor maximisation is equivalent to minimisation of the following expression

$$\frac{\int_0^T (\dot{u}^{\mathrm{T}}(t)\dot{u}(t))dt}{\max\limits_{0\le t\le T} (\dot{u}^{\mathrm{T}}(t)\dot{u}(t))} . \tag{2}$$

For notational convenience, let us introduce:

$$v(t) := \dot{u}(t) \tag{3}$$

and

$$x_{n+1}(t) := u(t) . \tag{4}$$

The above substitution can be interpreted as introducing the new state equation in the form

$$\dot{x}_{n+1}(t) = v(t), \quad x_{n+1}(0) = \xi_1 , \tag{5}$$

where ξ_1 is a parameter to be optimised.

Then the plant friendliness index (2) can be modified as follows

$$\frac{\int_0^T (v^{\mathrm{T}}(t)v(t))dt}{\max\limits_{0\le t\le T} (v^{\mathrm{T}}(t)v(t))} \tag{6}$$

The above problem can be suitably modified by defining the plant friendliness index as:

$$\frac{1}{c}\int_0^T \left(v^{\mathrm{T}}(t)v(t)\right)dt \tag{7}$$

subject to

$$\max_{0 \le t \le T} \left(v^{\mathrm{T}}(t) v(t) \right) \le c \,. \tag{8}$$

Introducing another state variable $x_{n+2}(t) = 1/c$ where $\xi_2 := 1/c$, we have

$$\dot{x}_{n+2}(t) = 0,\ x_{n+2}(0) = \xi_2 \,. \tag{9}$$

Then the problem (7) is equivalent to the canonical optimal control problem which maximises the performance index

$$\int_0^T x_{n+2}(t)\,(v^{\mathrm{T}}(t) v(t))\,dt \,, \tag{10}$$

along with the inequality constraint

$$x_{n+2}(t)\,(v^{\mathrm{T}}(t) v(t)) \le 1,\ \ t \in [0, T] \,. \tag{11}$$

The most suitable form to represent our problem is the Bolza functional form of the performance index, i.e. the sum of the function of terminal values of state variables and the integral of another function over the control period. That form of a performance index may be then expressed in the Lagrange functional form with a modified set of constraints. Such a typical form of the optimisation task allows obtaining the trajectory of the plant friendly input signal with the use of one of existing software packages for solving numerically optimal control problems.

3 The D-efficiency constraint formulation

The input signal employed in the identification experiment should simultaneously yield two results: the acceptable accuracy of parameter estimates and as spherical as possible the ellipsoidal confidence region of the estimates. Such a compromise can be reached applying an approach, which relies on the notion of the D-efficiency [15]. Any optimality criterion can be associated with the efficiency function, defined as a measure of the relative performance of any given experiment e compared to that of the optimal experiment e*. The D-efficiency, which may be considered as a measure of the D-suboptimality of given input (plant friendly) trajectories, is specified by

$$E_D(\mathbf{e}) = \left\{ \frac{\det(\mathbf{M}(\mathbf{e}))}{\det(\mathbf{M}(\mathbf{e}^*))} \right\}^{1/k} , \tag{12}$$

where k is the number of parameters to be identified, and $\mathbf{e}^*$ stands for the D-optimal trajectories which can be determined earlier. Following the reasoning and derivations presented in [16], we set a reasonable positive threshold $\eta < 1$ and impose the constraint on the D-efficiency value:

$$E_D(\mathbf{e}) \ge \eta \,. \tag{13}$$

Such an approach will yield a D-suboptimal, yet reasonable solution. The inequality (13) is equivalent to the constraint

$$\Psi[\mathbf{M}(\mathbf{e})] \le D\,, \tag{14}$$

where $D = \Psi\left[\mathbf{M}\left(\mathbf{e}^*\right)\right] - k\log(\eta)$.

The objective of such an experiment is formulated through maximisation of the FIM determinant (D-optimality) with respect to D-efficiency constraint (14). However if we assume a certain loss of D-optimality, we can use the equality form of the constraint (14).

4 Optimal input design for an inertial system design with respect to the cost function D-efficiency constraint

To illustrate the properties of the above approach to parameter identification, using the plant friendly input signal and with respect to the assumed level of D-optimality, we have selected a simple first-order (inertial) object. The transfer function of an inertial model has the following form

$$G(s) = \frac{k_p}{Ts+1}\,. \tag{15}$$

The problem of synthesising a plant-friendly input signal with respect to the D-efficiency constraint for an inertial system can be described by the following single input, single output state space model

$$\begin{aligned} \dot{x}(t) &= ax(t) + bu(t),\ x(0) = x_0 \\ y(t) &= x(t) \end{aligned}\,, \tag{16}$$

where x = x(t, a, b) and model parameters a and b are constant. The principle of the design of optimal input signals for system identification is to maximise the sensitivity of the state variable or the observation to the unknown parameter [3]. The justification for this approach is the Cramer-Rao lower bound, which provides a lower bound for the estimation error covariance. Providing this feature of the input, we obtain the parameter estimate or observation sensitivity which tends to be lowered for an optimal input

$$\operatorname{cov}([a,b]) \ge \mathbf{M}^{-1}\,. \tag{17}$$

The Fisher information matrix for the inertial model (16) can be expressed as

$$\mathbf{M}(T) = \int_0^T \begin{bmatrix} x_a \\ x_b \end{bmatrix} \begin{bmatrix} x_a & x_b \end{bmatrix} dt = \int_0^T \begin{bmatrix} x_a^2 & x_a x_b \\ x_a x_b & x_b^2 \end{bmatrix} dt = \begin{bmatrix} \int_0^T x_a^2 dt & \int_0^T x_a x_b dt \\ \int_0^T x_a x_b dt & \int_0^T x_b^2 dt \end{bmatrix}, \tag{18}$$

where $x_a = \partial x / \partial a$ and $x_b = \partial x / \partial b$. The problem can be suitably modified by defining the augmented state vector as [3]

$$\dot{x}_a = x + a x_a, \;\; x_a(0) = 0\,, \tag{19}$$

$$\dot{x}_b = a x_b + u, \;\; x_b(0) = 0\,. \tag{20}$$

A plant-friendly input signal for the inertial system perturbation is formulated through maximisation of the Fisher information matrix determinant (D-optimality) and the plant friendliness index maximisation in the form of a conventional integral-criterion optimal control problem. The problem of synthesising an optimal input signal for an inertial system, utilising Mayer's canonical formulation of the performance index, has been solved in literature [13, 14]:

$$\mathbf{M}(t) = \int_0^t \begin{bmatrix} x_a \\ x_b \end{bmatrix} \begin{bmatrix} x_a & x_b \end{bmatrix} d\tau \;\; // \;\; \frac{d}{dt}\,. \tag{21}$$

The FIM can be modified as follows:

$$\dot{\mathbf{M}}(t) = \begin{bmatrix} x_a \\ x_b \end{bmatrix} \begin{bmatrix} x_a & x_b \end{bmatrix}, \;\; \mathbf{M}(0) = 0\,, \tag{22}$$

where

$$\mathbf{M}(t) = \begin{bmatrix} m_{11}(t) & m_{12}(t) \\ m_{21}(t) & m_{22}(t) \end{bmatrix}, \tag{23}$$

and

$$\dot{m}_{11}(t) = x_a^2(t), \;\; m_{11}(0) = 0\,, \tag{24}$$

$$\dot{m}_{12}(t) = \dot{m}_{21}(t) = x_a(t) x_b(t), \;\; m_{12}(0) = 0\,, \tag{25}$$

$$\dot{m}_{22}(t) = x_b^2(t), \;\; m_{22}(0) = 0\,. \tag{26}$$

The augmented state equations, taking into account input friendliness factor formulation, are given by

$$
\begin{array}{lll}
x_1 = x, & \dot{x}_1 = ax_1 + bu, & x_1(0) = x_{10} \\
x_2 = x_a, & \dot{x}_2 = x_1 + ax_2, & x_2(0) = 0 \\
x_3 = x_b, & \dot{x}_3 = ax_3 + u, & x_3(0) = 0 \\
x_4 = m_{11}, & \dot{x}_4 = x_2^2,\ x_4(0) = 0 & \\
x_5 = m_{12} = m_{21}, & \dot{x}_5 = x_2 x_3, & x_5(0) = 0 \\
x_6 = m_{22}, & \dot{x}_6 = x_3^2, & x_6(0) = 0 \\
x_7 = u, & \dot{x}_7 = v, & x_7(0) = \xi_1 \\
x_8 = \frac{1}{c}, & \dot{x}_8 = 0, & x_8(0) = \xi_2.
\end{array}
\tag{27}
$$

Then the equivalent optimal control problem utilising Mayer's canonical formulation, which maximises the performance index with respect to the D-efficiency equality constraint, is

$$
J = \left(x_4(T)x_6(T) - x_5^2(T)\right) - \mu \int_0^T x_8(t)\,(v^{\mathrm{T}}(t)v(t))dt\,, \tag{28}
$$

subject to

$$
\begin{array}{l}
x_8(t)\,(v^{\mathrm{T}}(t)v(t)) \le 1,\ \ t \in [0,T] \\
\left(x_4(t)x_6(t) - x_5^2(t)\right) = D \\
-1 \le x_7(t) \le 1 \\
-10^4 \le v(t) \le 10^4
\end{array}
\quad , \tag{29}
$$

where μ is the input friendliness factor constant and D is D-efficiency constant.

Note that in the above formulation $v(t)$ (which is a derivate of the original control signal $u(t)$) acts as the input signal to the augmented system. We assumed the relatively wide range of the variability of $v(t)$ to enable abrupt changes of the original control signal $u(t)$, which is limited to the range of [-1, +1].

5 Experimental results for inertial case study

The above problem can be solved using one of the existing packages for solving dynamic optimisation problems, such as RIOTS_95 [17], DIRCOL [18] or MISER [19]. All computations were performed using low-cost PC (Atom, 1.66 GHz, 1 GB RAM) running Windows 7 and Matlab 7.5 (R2007b). Optimal and sub-optimal signals are computed for nominal values of parameters a = -1, b = 1 and assumed termination time T_f = 10 sec using SQP algorithm. The system is assumed to be at an initial state $x_1(0)$ = 5, the initial value of the input signal is $u(0)$ = 1 and -1 ≤ $u(t)$ ≤ 1. The system dynamics was integrated using the fixed step-size fourth-order Runge-Kutta method with grid intervals of 0.2 sec. The D-optimal input signals obtained for different values of the D-efficiency constant D and the maximum value of the input friendliness factor μ are shown in Figure 1, and Figure 2 (i.e. the maximum value of the input friendliness factor constant μ = 50 means that the corresponding D-optimal input signal is practically the same as the step input signal).

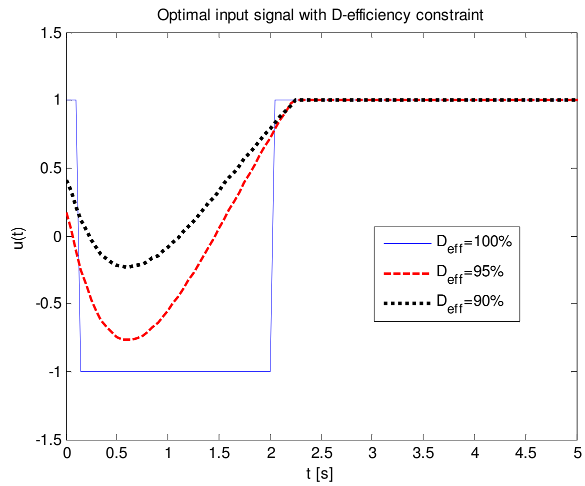

Fig. 1. Optimal and suboptimal input signals to the inertial system for different D-efficiency values

Rys. 1. Optymalny i suboptymalne sygnały sterujące układem inercyjnym dla różnych wartości D-efektywności

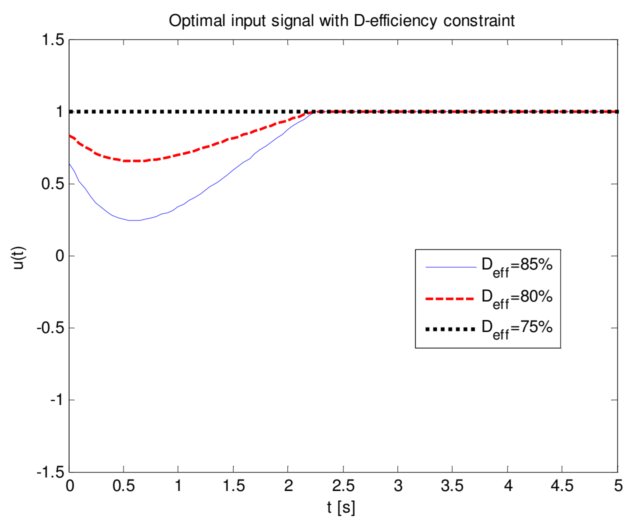

Fig. 2. Suboptimal input signals to the inertial system for different D-efficiency values

Rys. 2. Suboptymalne sygnały sterujące układem inercyjnym dla różnych wartości D-efektywności

In order to avoid getting stuck in a local minimum, all computations were repeated several times from different initial conditions. Each run took about forty minutes. The D-optimal excitation signal obtained when there was no constraint on the plant friendliness component (i.e. for $\mu \cong 0$) and the maximum value of the FIM determinant (i.e. for $D_{eff} = 100\%$) is shown in Figure 1. The input friendliness factor was increased to the maximum value (Figure 1 and Figure 2) to obtain the plant-friendly input signal with D-efficiency from the interval $75\% \le D_{eff} \le 100\%$. Figure 2 contains the graphical display of the sub-optimal input signal obtained for $\mu = 50$ and $D_{eff} = 75\%$, which is friendly for inertial system identification.

6 Conclusions

An optimal input signal design for system identification was formulated and the methods of the problem solution were outlined. The results of plant-friendly input signal design with guaranteed *D*-efficiencies for the inertial case study were presented. The D-optimal input signals obtained for different values of the D-efficiency constant *D* and the maximum value of the input friendliness factor μ still guarantee a reasonably small volume of the confidence ellipsoid for the estimates. Of significant importance is the fact that the proposed formulation can be transcribed into an equivalent optimal control problem in the Mayer form, and that it can be then solved using one of the existing packages for solving dynamic optimisation problems.

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Summary

The purpose of the current work is to formulate the optimization problem for plant friendly input signal design with respect to D-efficiency constraint. The objective of this kind of experiment design is to minimise the variance of the parameters to be estimated and to maximise the input signal friendliness. Since a plant friendly input signal does not necessarily guarantee richer information content in the measurements, an additional constraint is imposed on D-efficiency of the solution.

Keywords: plant friendly identification, D-optimality, D-efficiency

Dobór optymalnego sygnału w zadaniu estymacji parametrów układu inercyjnego z ograniczeniem na D-efektywność

Streszczenie

W niniejszej pracy sformułowano problem doboru przyjaznego sygnału wejściowego z ograniczeniem na D-efektywność funkcjonału celu. Celem takiego eksperymentu jest minimalizacja wariancji estymowanych parametrów oraz maksymalizacja przyjazności sygnału wejściowego. Stwierdzono, iż przyjazny sygnał sterujący nie zawsze gwarantuje uzyskanie akceptowalnych wartości estymat parametrów identyfikowanego modelu. Z tego powodu nałożono dodatkowe ograniczenie na D-efektywność funkcji celu.

Słowa kluczowe: identyfikacja przyjazna obiektowi, D-optymalność, D-efektywność